Quantum thermodynamics as a gauge theory

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Main idea

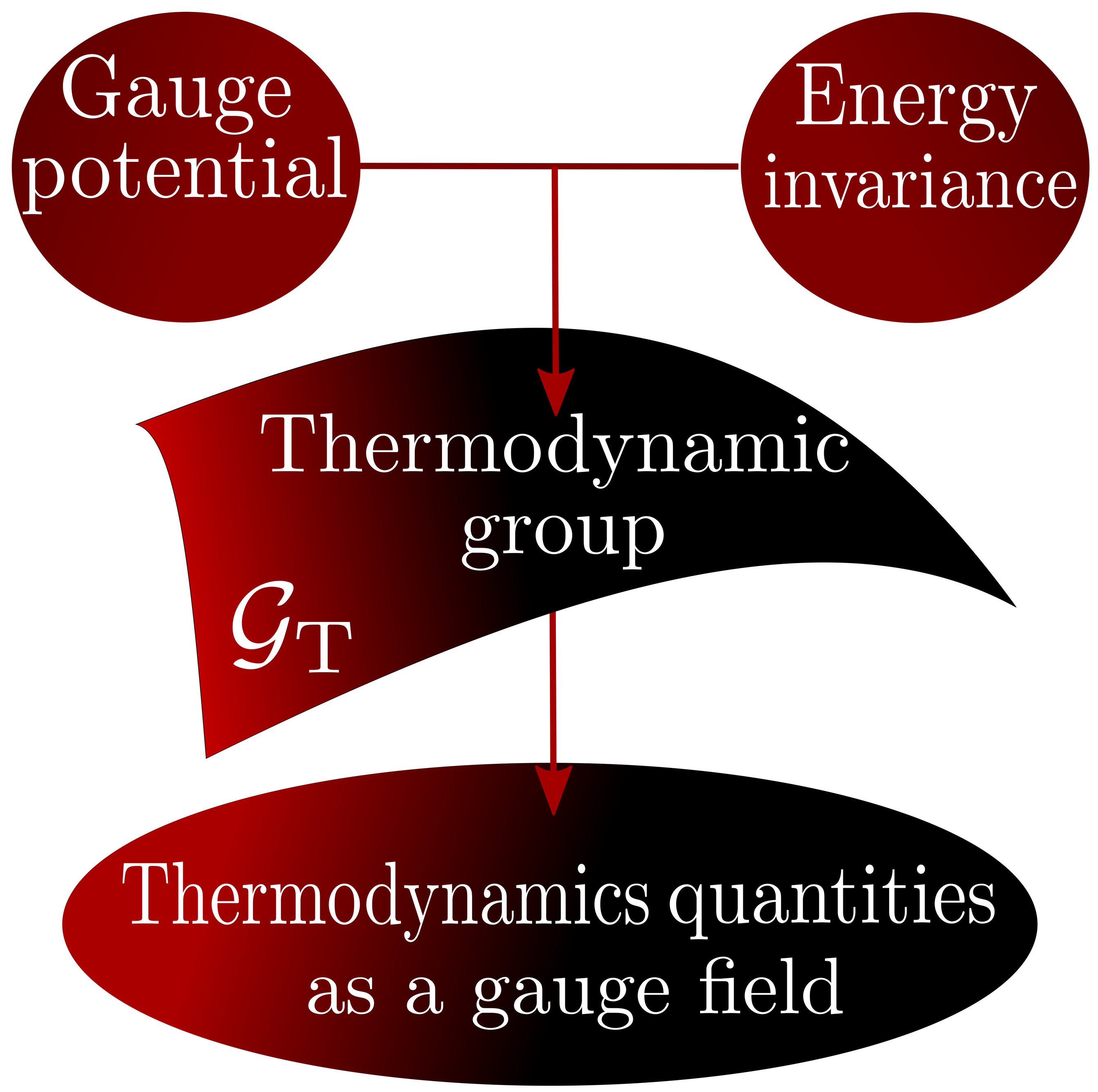
Thermodynamics is formulated through a coarse-grained approach, wherein fundamental variables emerge while effectively disregarding microscopic details. In contrast, quantum mechanics describes the dynamics of microscopic systems, seeking to predict experimental outcomes—a goal common to all fundamental physical theories, often formulated as gauge theories in modern physics.

Here, we formulate quantum thermodynamics as a gauge theory. Specifically, we develop the notions of work, heat, and entropy, which establish the foundation for gauge-invariant versions of the first and second laws.

Our results reveal the profound role of quantum coherences in our limited access to information about certain observables, which is quantified by heat production and entropy generation.

$\mathcal{G}_{\mathrm{T}}$-Quantum thermodynamics

To develop this formulation, we start from an analogy between the fundamental objects of quantum mechanics and thermodynamics and those of a gauge theory.



$\mathcal{G}_{\mathrm{T}}$-thermodynamics. The density operator acts as a gauge potential, with the Hamiltonian's internal symmetries constrained by average energy invariance. This defines the thermodynamic group $\mathcal{T}_{H_t}$, formed by all unitaries V_t commuting with H_t. The Lie group $\mathcal{G}_{\mathrm{T}}$, isomorphic to $\mathcal{T}_{H_t}$ is built and connect to the irreps of each k-eigenspace of H_t, inducing the Haar measure $\mathrm{d}\mathcal{G}_{\mathrm{T}}$. Physical quantities, treated as a gauge fields, emerges through Haar averaging over $\mathcal{G}_{\mathrm{T}}$

$$F_{inv}[\rho_t] = \underbrace{\int \mathrm{d}\mathcal{G}_{\mathrm{T}} F\left[V_t \rho_t V_t^{\dagger}\right]}_{\text{If } F \text{ not is unitary-invariant}} \text{ or } F_{inv}[\rho_t] = \underbrace{F\left[\int \mathrm{d}\mathcal{G}_{\mathrm{T}} V_t \rho_t V_t^{\dagger}\right]}_{\text{If } F \text{ is unitary-invariant}}.$$

In the context of the first law, we obtain gauge fields linked to invariant work and heat. The Haar average procedure reveals that changes in populations correspond to work, while changes quantum coherence in the energy eigenbasis correspond to heat which can be expressed as

$$Q_{inv}[\rho_t] = \underbrace{Q_u}_{\text{Alicki heat}} + \underbrace{Q_c}_{\text{Coherent heat}} + \underbrace{Q_{deg.}}_{\text{Degeneracys in } H_t}$$

For the second law, we show that $\mathcal{G}_{\mathrm{T}}$-entropy splits into two contributions:

$$S_{\mathcal{G}_{\mathrm{T}}}[\rho_t] = \underbrace{S_d[\rho_t]}_{\text{Diagonal entropy}} + \underbrace{S_{\Gamma.}}_{\text{Holevo asymmetry}}$$

In particular, we prove that S_Γ is entirely linked to degeneracies in H_t. Our numerical results in the LMG model show that, in the thermodynamic limit, asymmetry contributions vanish, establishing S_Γ as a classical-quantum marker and S_d arises as the entropy of this theory.

Messages

Changes in quantum coherences in the energy eigenbasis and degeneracies generate quantum heat and entropic uncertainty.

We establish an intrinsic relationship between the quantum coherence production, Hamiltonian degeneracies, and limited information about observables.

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