Relativistic quantum fluctuation theorem

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Main idea

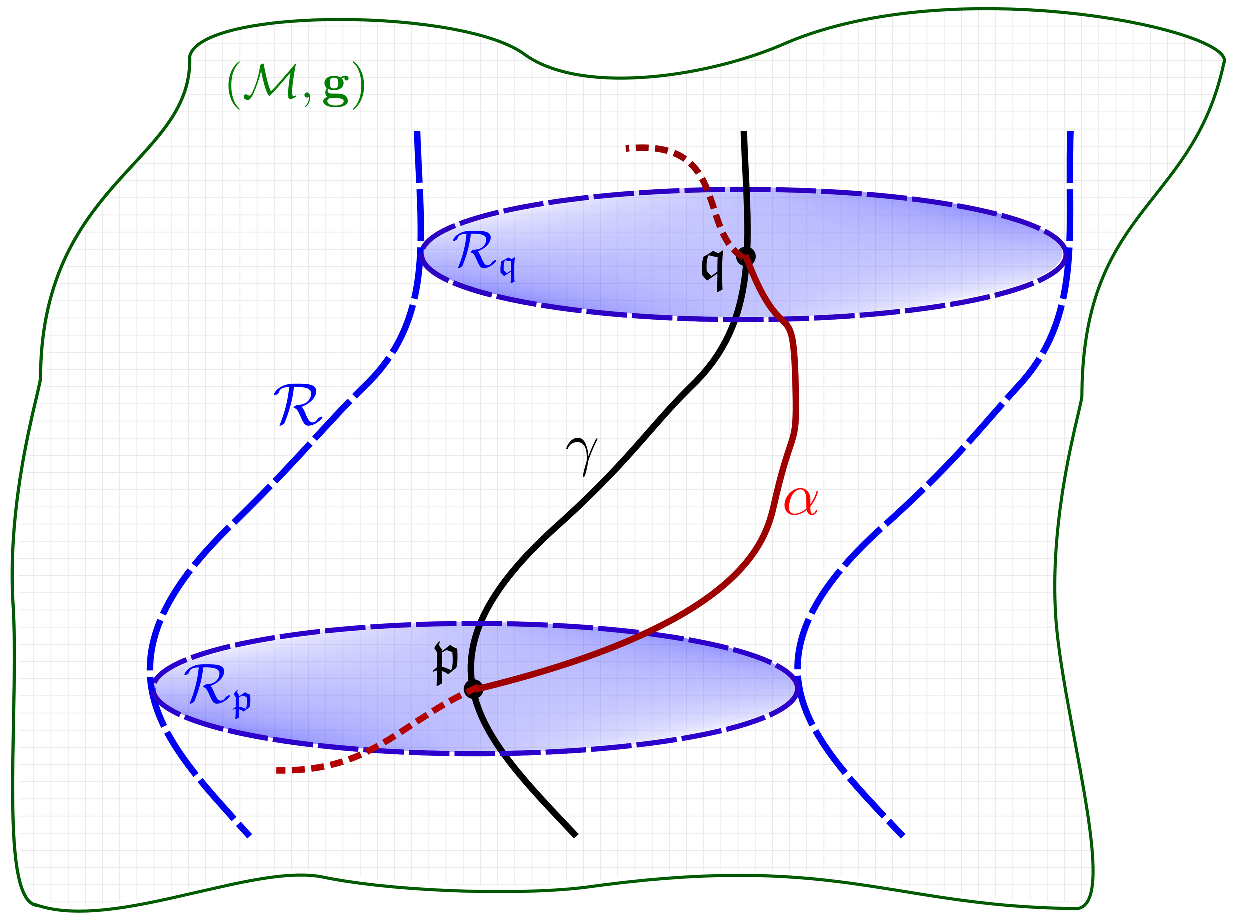
Irreversibility, marked by the production of thermodynamic entropy, establishes the thermodynamic arrow of time, pointing from low to high entropy. A significant step in this research area has been the formulation of fluctuation theorems.

A cloudy direction is the intersection of relativity and thermodynamics, that dates back to over a century ago. Despite several developments, a widely accepted theory is still lacking.

Here we prove a relativistic detailed fluctuation theorem for a quantum system under the action of gravity. Our result unveils the complete impact of the gravitational field on irreversible processes by explicitly considering the effect of spacetime curvature.

Sketch of the proof

We first build the Fermi normal coordinates around a time-like trajectory that describes the worldline of our laboratory frame (γ). Then, we consider the Hamiltonian formulation of the dynamics of a localized quantum particle around this time-like trajectory (that follows trajectory α). After this we employ the two-point measurement protocol (defined locally) in order to prove the theorem.



Sketch of the Fermi normal coordinate system. $\mathcal{R}$ is the region where the local rest spaces $\mathcal{R}_\mathfrak{r}$, for any $\mathfrak{r} \in \mathcal{M}$, are defined. The Fermi normal coordinate system (τ, x^1, x^2, x^3) covers each one of these subspaces. The system world-line $\alpha \subset \mathcal{R}$ and the laboratory world-line γ are also shown.

Specifically, the Hamiltonian defines a notion of time flow, which is crucial for defining local thermal equilibrium states. We consider two protocols: the forward direction of time and the backwards direction. The distinguishability of these processes is then employed as a measure of irreversibility of the forward process. Both processes consist in preparing the system in an equilibrium state, measuring its energy, letting it evolve and measuring its final energy. Work is defined as the difference between the results of these measurements. The forward and backward work probability distribution fulfils the detailed fluctuation theorem

$$\frac{P_{\text{fwd}}(W)}{P_{\text{rev}}(-W)} = e^{\beta(W - \Delta F)},$$

with ΔF being the difference in free energy. All the above quantities are fundamentally dependent on the spacetime curvature. For instance, in the case of a harmonic oscillator living in an expanding universe, the transition probability between two energy eigenstates (k and l), considering an expansion time τ is proportional to the Hubble constant $\mathbb{H}$

$$p^{\tau}_{k|l} \propto 4\left(\frac{m\mathbb{H}^2}{2}\right)^2$$

thus leading to a positive entropy production.

Messages

Entropy production is not an invariant quantity defined solely by the system.

We established a deep and fundamental link between the time-orientability of spacetime and the thermodynamic arrow of time.

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