**Institute of Physics — Federal University of Goiás**

**Quantum Pequi Group**

# General Relativity

# Lecture Notes

**Lucas Chibebe Céleri**

Goiânia, 2025

# Notation and Convention

Before we begin, let us clarify some notations and conventions. First, we use the Einstein summation convention, where repeated indices are implicitly summed over:

$$\sum_a \mathrm{x}^{\mathrm{a}} \mathrm{y}_{\mathrm{a}} \equiv \mathrm{x}^{\mathrm{a}} \mathrm{y}_{\mathrm{a}}.$$

Moreover, we employ the abstract index notation, in which Latin indices $\{a, b, c, ...\}$ in tensors do not represent their basis components, but only their slots. In this notation, a tensor $\mathrm{T}$ of rank $(k, l)$ will be denoted by

$$\mathrm{T} \equiv \mathrm{T}^{\mathrm{a}_1, \ldots, \mathrm{a}_\mathrm{k}}{}_{\mathrm{b}_1, \ldots, \mathrm{b}_\mathrm{l}}.$$

Therefore, the indices denote how many contravariant ($k$) and how many covariant ($l$) slots define the tensor $\mathrm{T}$. This is very important since we need to distinguish between equations that hold in a specific coordinate system, which are written in terms of the components of the tensors in that system, and equations that are independent of the choice of basis, which are the ones written in terms of the abstract index notation.

To write down the equations for the components of the tensor for which a specific basis is required, we employ a Greek set of indices. In this way, $T^{\alpha\beta\gamma}{}_{\delta\epsilon}$ are the components of the tensor $\mathrm{T}^{\mathrm{abc}}{}_{\mathrm{de}}$ in some specific coordinate system. In this sense, latin indices denote the number and type of variables the tensor acts on, and not its coordinate components.

It is also convenient to introduce a notation for the totally symmetric

$$\mathrm{T}_{(\mathrm{ab})} = \frac{1}{2}\left(\mathrm{T}_{\mathrm{ab}} + \mathrm{T}_{\mathrm{ba}}\right)$$

and the totally anti-symmetric

$$\mathrm{T}_{[\mathrm{ab}]} = \frac{1}{2}\left(\mathrm{T}_{\mathrm{ab}} - \mathrm{T}_{\mathrm{ba}}\right)$$

parts of a tensor.

Note that we also employed the notation $\mathrm{T}$ to represent a tensor, while $T$ is used for its components. This notation will also be employed for vectors. In this way, $\mathrm{x}$ or $\mathrm{x}^{\mathrm{a}}$ represents the coordinate vector, while $x^\mu$ stands for its coordinates in a specific reference frame. Three-vectors in space will be denoted by a letter with an arrow, like $\vec{x}$.

We also employ metric signature $(+,-,-,-,)$ and natural units in which $G$ (Newton's gravitational constant), $c$ (the speed of light), $k$ (Coulomb's constant), $k_B$ (Boltzmann constant) and $\hbar$ (Planck's constant) are set to one.

Sets will be denoted by a letter containing a double line as in $\mathbb{V}$, like the sets of vectors in a vector space.

# Contents